Practice Exam 2

Fall 2011

Name: ______________________ Date: ____________

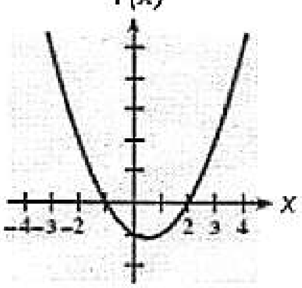
1. Both a function and its derivative are given. Use them to find the relative maxima.

$f(x)=x-6x^{2/3}+9 \quad f'(x)=\dfrac{x^{1/3}-4}{x^{1/3}}$

A) $(0,9)$
B) $(-23,64)$
C) $(64,-23)$
D) $(0,9),(64,-23)$
E) no relative maxima

2. A graph of $f'(x)$ is given. Use the graph to determine where $f(x)$ is decreasing.

$f'(x)=3x^2-3x-6$



A) $x>-1$
B) $x<2$
C) $x>2$
D) $x<-1$ or $x>2$
E) $-1<x<2$

3. Make a sign diagram for the function and determine all x-values at which relative maxima occur.

$y=x^3-3x^2-45x+1$

A) $x=0$
B) $x=1$
C) $x=5$
D) $x=-3$
E) no relative maxima

4. A function and its first and second derivatives are given. Use these to find all points of inflection.

$y = x^{4/3}(x-7)+6$

$y' = \dfrac{7x^{1/3}(x-4)}{3}$

$y'' = \dfrac{28(x-1)}{9x^{2/3}}$

A) $(0,6)$

B) $(1,0.000)$

C) $(4,-13.049)$

D) $(4,-23.049)$

E) no points of inflection

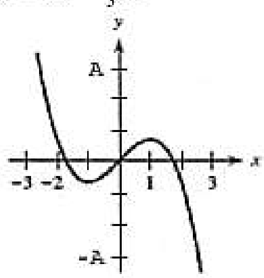
5. For the given function, find intervals of x-values where the function is decreasing.

$y = \dfrac{x^4}{4} - \dfrac{x^3}{3} - 9$

A) $0 < x < 1$
B) $x > 0$
C) $x < 0$
D) $x > 1$
E) $x < 1$

6. For the given function and graph, estimate the coordinates of the relative maxima by observing the graph, where $A = 6$.

$y = 2x - \frac{2}{3}x^3$



A) $(1, \frac{2}{3})$

B) $(1, \frac{4}{3})$

C) $(-1, -\frac{4}{3})$

D) $(-1, -\frac{2}{3})$

E) no relative maxima

7. For the given function, find $y' = f'(x)$.

$$y = \frac{x^4}{4} - \frac{x^3}{3} - 2$$

A) $x^3 - x$
B) $x^4 - x^2 - 2$
C) $x^3 - x^2 - 2$
D) $x^4 - x^3$
E) $x^3 - x^2$

8. ***Oxygen purity—diminishing returns*** Suppose that the oxygen level P (for purity) in a body of water t months after an oil spill is given by $P(t) = 400\left[1 - \frac{2}{t+2} + \frac{4}{(t+2)^2}\right]$. Find how long it will be before the rate of change of P is maximized. That is, find the point of diminishing returns.

A) $t = 0$
B) $t = 2$
C) $t = 4$
D) $t = \inf$
E) none of the above

9. For the given function, find the critical values.

$$y = \frac{x^4}{4} - \frac{x^3}{3} - 4$$

A) $x = 0$ and $x = 1$
B) $x = 0$ and $x = 4$
C) $x = 0$ and $x = -4$
D) $x = 0$ and $x = -1$
E) $x = -1$ and $x = 1$

10. A function and its first and second derivatives are given. Use these to find the relative maxima.

$f(x) = x^5 - \mathrm{a}\, x^4 + \mathrm{b}$

$f'(x) = 5x^3(x - 4/5 * \mathrm{a})$

$f''(x) = 20x^2(x - 3/5 * \mathrm{a})$

A) $(0, \mathrm{b})$

B) $(3/5 * \mathrm{a}, \mathrm{inf})$

C) $(\mathrm{minx}, \mathrm{miny})$

D) $(\mathrm{minx}, -\mathrm{miny})$

E) no relative maxima

11. Both a function and its derivative are given. Use them to find the relative minima.

$f(x) = x - 6x^{2/3} + 8 \qquad f'(x) = \dfrac{x^{1/3} - 4}{x^{1/3}}$

A) $(0,8)$

B) $(-24,64)$

C) $(64,-24)$

D) $(0,8),(64,-24)$

E) no relative minima

12. A function and its first and second derivatives are given. Use these to find all relative minima.

$f(x) = x^5 - \mathrm{a}\, x^4 + \mathrm{b}$

$f'(x) = 5x^3(x - 4/5 * \mathrm{a})$

$f''(x) = 20x^2(x - 3/5 * \mathrm{a})$

A) $(0, \mathrm{b})$

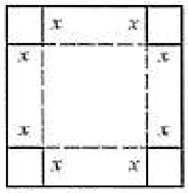
B) $(3/5 * \mathrm{a}, \mathrm{inf})$

C) $(\mathrm{minx}, \mathrm{miny})$

D) $(0, \mathrm{b}),\ (\mathrm{minx}, \mathrm{miny})$

E) no relative minima

13. ***Volume*** A rectangular box with a square base is to be formed from a square piece of metal with 36-inch sides. If a square piece with side x is cut from each corner of the metal and the sides are folded up to form an open box, the volume of the box is $V=(36-2x)^2x$. What value of x will maximize the volume of the box?



A) 18
B) 7
C) 6
D) 17
E) 2

14. p is in dollars and q is the number of units. Find the elasticity of the demand function $pq=63$ at $p=\$5$.
A) –1.00
B) –0.08
C) –12.60
D) 1.00
E) 12.60

15. ***Compound interest*** If \$7500 is invested at an annual rate of 12.5% compounded continuously, the future value S at any time t (in years) is given by $S=7500e^{0.125t}$. What is the amount after 18 months?
A) \$71,158.02
B) \$9037.96
C) \$40,102.37
D) \$8906.25
E) \$9046.73

16. Solve the exponential equation. Give answers correct to 3 decimal places.

$7^{6x}=343$
A) 49
B) 0.500
C) 0.565
D) 0.294
E) 25

17. ***Supply*** Suppose that the supply of x units of a product at price p dollars per unit is given by $p = 40 + 70\ln(5x+2)$. Find the rate of change of supply price with respect to the number of units supplied.

A) $\frac{dp}{dx} = \frac{70}{5x+2}$

B) $\frac{dp}{dx} = \frac{5}{5x+2}$

C) $\frac{dp}{dx} = \frac{4900}{5x+2}$

D) $\frac{dp}{dx} = \frac{350}{5x+2}$

E) $\frac{dp}{dx} = \frac{140}{5x+2}$

18. Find the derivative of the following function.

$y = \ln(9x^3 - 7x) - 9x$

A) $\frac{27x^2 - 7x}{x(9x^2 - 7)} - 9$

B) $\frac{1}{x(9x^2 - 7)} - 9$

C) $\frac{27x^2 - 7}{x(9x^2 - 7)} - 9$

D) $\frac{27x^2}{x(9x^2 - 7)} - 9$

E) $\frac{1}{x(9x^2 - 1)} - 9$

19. ***Decibels*** The loudness of sound (L, measured in decibels) perceived by the human ear depends on intensity levels (I) according to $L = 10\log_{10}(I / I_0)$, where I_0 is the standard threshold of audibility. If $x = I / I_0$ then using the change-of-base formula, we get $L = \dfrac{10\ln(x)}{\ln 10}$. At what rate is the loudness changing with respect to x when the intensity is 1000 times the standard threshold of audibility (that is, when x = 1000)?

A) $\dfrac{1}{1000 \ln 10}$

B) $\dfrac{1}{100 \ln 10}$

C) $\dfrac{1}{1000}$

D) $\dfrac{1}{100}$

E) $\dfrac{1}{50}$

20. ***Sales decay*** The sales decay for a product is given by $S = 300e^{-2t}$, where S is the daily sales in dollars and t is the number of days since the end of a promotional campaign. Find the rate of change of sales decay.

A) $\dfrac{dS}{dt} = -600e^{-2t}$

B) $\dfrac{dS}{dt} = 600e^{-2t}$

C) $\dfrac{dS}{dt} = -1200e^{-2t}$

D) $\dfrac{dS}{dt} = 1200e^{-2t}$

E) $\dfrac{dS}{dt} = -300e^{-2t}$

21. Find the derivative of the following function.
$y = \ln 4x$

A) $\frac{1}{x}$
B) $\frac{4}{x}$
C) $\frac{1}{4x}$
D) $\frac{1}{x^2}$
E) $\frac{1}{4x^2}$

22. Use properties of logarithms or a definition to simplify the expression. Check the result with a change-of-base formula and a calculator.

$\log_2 16$

A) 32.00
B) 4.00
C) 2.77
D) 1.20
E) No solution

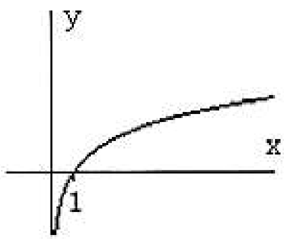
23. Write the expression as the sum or difference of two logarithmic functions containing no exponents.

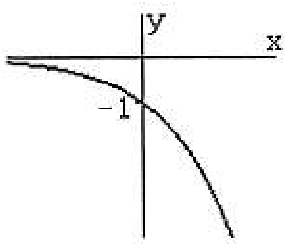
$\log_3(x\sqrt[3]{x+2})$

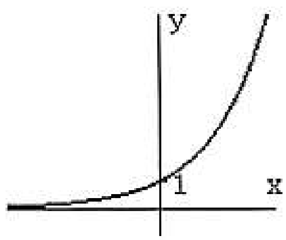
A) $3\log_3 x(x+2)$
B) $\frac{1}{3}\log_3 x(x+2)$
C) $\log_3 x + 3\log_3(x+2)$
D) $\log_3 x + \frac{1}{3}\log_3 x + \frac{1}{3}\log_3 2$
E) $\log_3 x + \frac{1}{3}\log_3(x+2)$

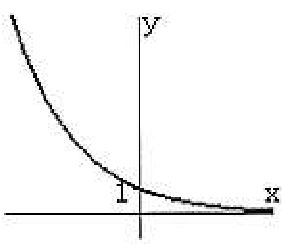
24. Graph the function.

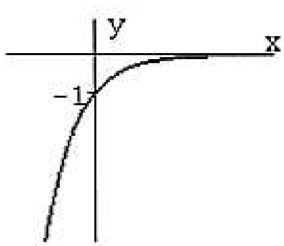
$y = 5^{-x}$

A) 

B) 

C) 

D) 

E) 

25. Calculate the logarithm $\log_2 2$ without using a calculator or tables.

A) 4
B) 1
C) 0
D) 2
E) -1

Answer Key

1. A
2. E
3. D
4. B
5. E
6. B
7. E
8. D
9. A
10. A
11. C
12. C
13. C
14. D
15. E
16. B
17. D
18. C
19. B
20. A
21. A
22. B
23. E
24. D
25. B